

ARTHUR M. DIAMOND, JR.

SCIENCE AS A RATIONAL ENTERPRISE

ABSTRACT. Scientists often disagree about whether a new theory is better than the current theory. From this some (e.g., Thomas Kuhn) have inferred that the values of science are changing and subjective, and hence that science is an 'irrational' enterprise. As an alternative, this paper develops a 'rational' model of the scientific enterprise according to which the scope and elegance of theories are important elements in the scientist's utility function. The varied speed of acceptance of new theories by scientists can be explained in terms of the optimal allocation of time among different scientific activities. The model thus accounts for the 'rationality' of science in a way that is broadly consistent with the empirical evidence on the history and practice of science.

INTRODUCTION

Scientific progress did not pose a problem for the logical positivists. Science progressed because scientists sought simple, general and rigorous explanations of phenomena. Science was defined by the universal, eternally valid values that scientists pursued. Scientific progress only became a problem when Thomas Kuhn (1962) highlighted historical cases of theory acceptance that seemed to show that scientists did not share common values.

Those who value science might safely ignore the problem of scientific progress if models of science had no impact on the actual practice of science. It is plausible, however, to think that the accepted model of science has a significant impact. One channel for such impact would be the attitudes of the educated lay public. Such attitudes are important because they effect the level of support, charitable, political and moral, that science receives. A second channel for the impact of the accepted model of science is through the scientist himself. The model of science that a person accepts will effect his occupational choice and, if he chooses science, his allocation of time and effort between science work and non-science-related leisure. The psychic returns from advancing science have long been identified as one of the primary components in the

Theory and Decision 24 (1988) 147-167.

© 1988 by D. Reidel Publishing Company.

compensation of scientists. Adam Smith even seems to suggest that such psychic returns are the *main* component of a scientist's compensation:

Mathematicians, ..., who may have the most perfect assurance, both of the truth and of the importance of their discoveries, are frequently very indifferent about the reception which they may meet with from the public. ... The great work of Sir Isaac Newton, his *Mathematical Principles of Natural Philosophy*, I have been told, was for several years neglected by the public. The tranquility of that great man, it is probable, never suffered, upon that account, the interruption of a single quarter of an hour (p. 124).

Even though we now know that Smith was wrong about Newton's tranquility (see: Hall, 1980, *passim* and Westfall, 1980, pp. 698–780), Smith may still be correct in his more general claim that scientists receive psychic returns from the belief that their research is true and important.

The problem that Kuhn raised of how to reconcile scientific progress with the actual history and practice of science is thus an important one for the reasons that I have just sketched. The attempt to solve the problem has touched off a debate over the proper criteria for evaluating models of science. It is generally agreed that logical positivism is flawed by its failure to deal with the history of science, while Kuhn's position is flawed by its failure to account for scientific 'rationality' (note that when scientists and philosophers refer to scientific 'rationality', they are using a stronger sense of 'rationality' than the mere consistency usually meant by the economist when he uses the word).

The first criterion for evaluating a model of science is thus that it accounts for our maintained prior belief that science is 'rational' in a stronger sense than that its practitioners are consistent. The precise definition of 'rationality' in science is still a topic of live dispute among philosophers and scientists. For a model of the behavior of scientists to account for the 'rationality' of science does not, however, require that this dispute be resolved, but only that the model imply behavior that is consistent with commonly accepted historical examples of rationality in science (Laudan, 1977, p. 160). Any acceptable model of scientific rationality would have to imply, for instance, that a scientist's acceptance of astrology in 1985 would have been irrational.

The second criterion for judging a model of science is that it explains the stylized facts from the history of science about the actual behavior of scientists. There may be several such stylized facts that are intrinsically worthy of note. But so far only one has been given significant attention:

that younger scientists tend to accept new theories more readily than older scientists. Max Planck gave the most vivid and best known description of this phenomenon when he said:

... a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.¹

Kuhn and others such as Lucas and Sargent (Klamer, 1983, pp. 35, 49–50, 70) have affirmed Planck's principle.

The interpretation of Planck's principle implicit in Kuhn and others begins by assuming that the differential acceptance of theories by age cannot be a rational phenomenon and thus must be spoken of in terms borrowed from religion or politics. The argument proceeds with the observation that differential acceptance by age is universal for all scientific communities. The conclusion is then that science even at its best is an irrational activity.

Post-Kuhnian models of science have tried to both account for scientific progress (i.e., scientific rationality) and also remain relevant to the history of science. A critique of several of the more important of these models can be found in an earlier study (Diamond, 1978, pp. 10–37). In the earlier study I concluded that the post-Kuhnian models had failed, usually because they follow Kuhn in believing that the historical evidence requires them to deny universal, stable scientific values. In adhering to Kuhn in this, they are no better able than he to account for scientific progress (see also Stove, 1982). In particular, progress remains a puzzle in the accounts of the later Kuhn, Toulmin, Laudan, Shapere, Lakatos and Feyerabend. Stegmüller fails for a different reason: he follows the positivists in constructing complex, historically irrelevant formalisms.

Even more recently, McCloskey has criticized all models of science as being too "thin", opting instead for an analysis of the rhetoric of the discourse of scientists (McCloskey, 1986a, 1986b). McCloskey is on target in exposing the frequent divergence between how a scientist *actually* argues and the account given by the scientist and the philosopher of how the scientist *should* argue. But McCloskey's approach does not permit any demarcation between science and other intellectual enterprises. The key problem for McCloskey is then, why, if the argument methods of science and non-science are the same, have the results of scientific arguments

occasionally been so powerful? Or again, why does science advance and the humanities do not?

THE MODEL

A maximization-under-constraints model holds promise of being able to account for scientific progress in a way consistent with the history of science. Such models explain differences in behavior by claiming that, though all agents maximize the same utility function, they differ in the constraints that they face (Diamond, 1982). Using this sort of model to explain human behavior is called by Becker "the economic approach" (1976).

Central to my model of science is the claim that to be rational is to pursue certain values. In particular I claim that the rational scientist is a person who maximizes the scope and elegance of scientific theories in any given domain. In symbols, I claim that the rational scientist maximizes the following utility function:

$$(1) \quad U = \sum_{k=1}^D \sum_{i=1}^{\omega} f(S_{ki}, E_{ki})$$

where U is the scientist's utility, k is an index of scientific domain, D is the number of domains in science, i is the present time period, ω is the final time period, S_{ki} is the scope of the theory in domain k that has the greatest scope and elegance in period i , and E_{ki} is the elegance of the theory in domain k that has the greatest scope and elegance in period i .

By the scope of a theory I mean the range of phenomena that can be inferred from it. By the domain of a theory I mean the kind of phenomena that the theory explains. Thus a theory will explain all of the phenomena within its scope but only a part of the phenomena within its domain. The scope of a theory can be increased either by increasing the range of phenomena explained or by increasing the detail of the explanation of a constant range of phenomena. We will call the first sort of scope 'extensive scope' and the latter sort 'intensive scope'. In deciding which of two theories has greatest overall scope, it is sometimes necessary to compare their relative values of the two sorts of scope. For example, Newtonian physics was overall superior in scope to Aristotelian physics because the superiority of Newtonian physics in extensive scope far outweighed the superiority of Aristotelian physics in intensive scope.²

The elegance of a theory means the ease of application or directness of inference of the theory. Usually, but not always, the theory with the least analytic apparatus is the most elegant. The rare exceptional case may be illustrated by an example from logic. It has been shown³ that any logical statement using the connectives \sim , \vee , and $\&$ can be equivalently expressed using the single connective \downarrow . Unfortunately, statements expressed in terms of \downarrow , are much longer and more cumbersome than the equivalent expressions in terms of \sim , \vee , and $\&$. So \downarrow is seldom used. The system of notation using \sim , \vee , and $\&$ is, in our sense, more elegant than using \downarrow . If in housebuilding a new tool is developed that performed well the functions of both the screwdriver and the chisel, then this would be an advance in efficiency. But it is no advance to use a screwdriver as a chisel for the sole purpose of getting by with one tool.

The content of utility function (1) illustrates one – but only one – way in which the maximization-under-constraints approach might be applied to science (see Sarkar, 1983). One attractive alternative would be to claim that the scientist maximizes, not the scope and elegance of theories, but rather his own income and prestige (see: Diamond, 1984; and Grubel and Boland, undated). Since much of the behavior of non-scientists has been explained in terms of income and prestige maximization, this alternative has the advantage of generality. The disadvantage of the alternative is that it does not require that science be rational. Thus it leaves unexplained the fact of scientific progress. Of course it is possible that social institutions are so structured that those who most further the scope and elegance of theories will also be those who receive the most income and prestige.⁴ In such a case, the progress of science would be intelligible. But it would only be so because scientists acted *as if* they were maximizing the scope and elegance of theories. It would thus still be illuminating to posit scope and elegance-maximization in order to account for the rationality of science.

But, in fact, there is good reason to believe that scientists actually do value scope and elegance. The mechanism for rewarding fruitful scientific research is imperfect because ideas are costly to appropriate and because the ultimate fruits of ideas are costly to assess.⁵ Thus the reward for producing new ideas will be below the value of the marginal product of such ideas to society. Casual observations confirms that scientists earn less than those in other professions that have equal prestige and equal training costs. If so, then some other psychic return would have to

compensate the scientist for foregone money income. The natural candidate for such psychic return is the satisfaction derived from furthering the scope and elegance of theories.

Sociobiology may provide a deeper explanation of why some persons derive psychic satisfaction from advancing science. Just as kin groups with an altruistic member have a survival advantage over those without one, so too a society with some members who have scientific values may have a survival advantage over those societies without such members (Wilson, 1975, pp. 106–129; Becker, 1976, pp. 282–294; Hull, 1978).

Utility function (1) has three advantages worth mentioning briefly. The first is the compatibility of the utility function with the reflective judgments of scientists themselves and with the tenor of much of the pre-Kuhnian models of science. Thus the utility function incorporates old and well-known criteria for judging theories. The second advantage is the utility function's consistency with the fundamental economizing function of theories. Theorizing is a mental technology that allows us to increase the amount of information we can store and to speed up our access to what is stored. The more scope and elegance a theory has, the more successfully it will fulfil this function. The third advantage of the utility function is that it is consistent with science's function of promoting man's mastery over his world. The greater the scope and elegance of theories, the lower the mental costs of information storage and access. Information about the world is a primary factor in producing technological advance. Thus theories with greater scope and elegance will lower information costs and thereby lower the price of technological advance.

The main advantage of the utility function, however, is not any of the three just mentioned, but rather that the utility function when combined with appropriate constraints, forms a model that is capable of satisfying the two criteria for evaluating models of science: it is consistent with the rationality of science and it explains stylized facts about the actual behavior of scientists. I begin with a detailed account of the model that I propose for rational science. This model will include the utility function that has been the focus so far, but will also incorporate constraints in the form of production functions and a time constraint. The equilibrium conditions for maximizing the utility function will be presented and some implications of these conditions will be discussed. Finally, it will be shown that the model satisfies the criteria for evaluating models of science.

In order to maximize his utility each scientist wants to contribute as much to the advancement of science as possible. Two sorts of activity can contribute to the maximization of this function. A scientist can increase the scope and elegance of the best theory either by adding scope and elegance to the old theory or by discovering a new theory that has promise of surpassing the previously dominant one in scope and elegance. I label the fruits of the first activity N (for advancing a 'new' theory) and those of the second C (for advancing a 'current' theory).

N and C are intended to be defined broadly enough so that all scientific activity fits into one or the other. Most activities fit fairly comfortably. One seeming exception, however, is the testing of theories. It is only a seeming exception and not a real one because the aim of testing is to learn the scope of a theory. What is finally of value is not conjectured scope but only thoroughly tested scope. So testing will necessarily be a part of both N and C .

Assume that in an earlier stage of the optimization process the scientist has selected a domain in which to work. Then, in formal terms, the scientist's problem is to maximize the following:

$$(2) \quad V = \sum_{i=1}^{\omega} f(S_i, E_i) = \sum_{i=1}^{\omega} \theta(C_i, N_i)$$

where $\partial V/\partial C > 0$ and $\partial V/\partial N > 0$. The model is one of maximization under constraints. V is what is to be maximized. There are three constraints: two production functions and one budget (time) constraint.

Potentially, there is a wide range of separate inputs that might be candidates for inclusion in the production functions for N and C . Of the three commonly used factor inputs – time, money and effort – only the first will be treated explicitly in the model. The justification for this is that the inclusion of money and effort would have no systematic implications for the phenomena to be explained, viz., the allocation of time between work on new theories and work on the currently dominant theory. There is no good reason to suppose that work on new theories is either more or less effort and money intensive than work on the old theory. Thus the inclusion of effort and money would yield no insights about the phenomena to be explained.

The first constraint is the production function for N :

$$(3) \quad N_i = t_{N_i} \gamma \quad \text{for } i = 1, \dots, \omega$$

where t_{N_i} is the time spent producing N in period i , and γ is the productivity coefficient of time spent in producing N . γ will vary both with individual and with domain-wide factors. For instance, intelligence is an individual factor – the more intelligent the scientist, the larger his γ . Another individual factor would be the quality and research focus of the professors with whom the scientist studied in graduate school. On the other hand, the more ‘ripe’ the domain is for a new discovery, the higher will be the γ of all scientists working in the domain. Thus ‘ripeness’ is a domain-wide factor depending in part on anomalies in the old theory. We will assume for simplicity of exposition that γ and the other parameters in the model are time invariant. In a more complex model γ would probably decline with age since one factor that heavily influences the promise of a new theory or idea is the opportunity to elaborate and promulgate it. The costs of communicating a new idea or theory, especially at the early stages, are very high. Thus the ‘discovery’ work-in-progress of a scientist who dies is less likely to be picked-up than work he had been doing to extend the scope and elegance of the currently dominant theory.

A second constraint is the production function for C_i :

$$(4) \quad C_i = t_{C_i}^\delta \left(\beta + \varepsilon \sum_{j=0}^i \left(\frac{t_{C_j}}{(1+r)^j} \right) \right) \quad \text{for } i = 1, \dots, \omega$$

where t_{C_i} is the time the scientist spends in period i to add scope and elegance to the currently dominant theory, and δ is the productivity coefficient of time spent in producing C_i . The term in the innermost parentheses implies that time spent in extending the current theory is more productive, the more time the scientist has already devoted to extending the current theory. The term may be thought of as representing human capital in the current theory. Due to forgetfulness, human capital acquired in earlier periods would depreciate, hence the inclusion of the r parameter. The constant β is intended to represent the human capital that the scientist acquired in graduate school. The size of the coefficient ε indicates the importance for current productivity of time spent extending the current theory in earlier periods. Production functions (3) and (4) can be substituted into utility function (2) to obtain:

$$(5) \quad V = f\left(t_{N_i}^\delta, t_{C_i}^\delta \left(\beta + \varepsilon \sum_{j=0}^i \left(\frac{t_{C_j}}{(1+r)^j} \right) \right) \right) \quad \text{for } i=1, \dots, \omega$$

In addition to the two production functions, the third and final constraint in the model is a time constraint:

$$(6) \quad T_i = t_{N_i} + t_{C_i}$$

where T_i is the total amount of time the scientist spends in scientific activity in period i . T_i is assumed to be exogenously determined and to have the same value in every period.⁶ A scientist, by definition, is a person who contributes to the advance of science. So:

$$t_{C_i}^* + t_{N_i}^* > 0$$

But a scientist need not, in any given period, contribute both to advancing the current theory and to discovering a new one. Hence:

$$t_{C_i}^* \geq 0 \quad \text{and} \quad t_{N_i}^* \geq 0$$

If (5) and (6) are used to construct the Lagrangean function in the usual way (see, e.g., Silberberg, 1978, pp. 154–158), then the Kuhn-Tucker first-order conditions for a maximum are:

$$(7) \quad \frac{\partial L}{\partial t_{C_i}} = \frac{\partial V}{\partial t_{C_i}} - \lambda_i \leq 0 \quad \text{if } <, t_{C_i} = 0$$

$$(8) \quad \frac{\partial L}{\partial t_{N_i}} = \frac{\partial V}{\partial t_{N_i}} - \lambda_i \leq 0 \quad \text{if } <, t_{N_i} = 0$$

$$(9) \quad \frac{\partial L}{\partial \lambda_i} = T_i - t_{C_i} - t_{N_i} = 0$$

where L is the Lagrangean function and λ is the Lagrange multiplier which can be interpreted as the marginal utility of time. Since condition (9) is an equality we know that $\lambda > 0$. If both conditions (7) and (8) are equalities then we have the usual condition for an optimal allocation of time when there is an interior solution:

$$\lambda_i = \frac{\partial V}{\partial t_{C_i}} = \frac{\partial V}{\partial t_{N_i}}$$

The scientist would allocate some time both to advancing the current theory and to discovering a new theory.

If condition (7), but not condition (8), is an equality then we have a corner solution where:

$$\lambda_i = \frac{\partial V}{\partial t_{C_i}} > \frac{\partial V}{\partial t_{N_i}}$$

The scientist would only spend time advancing the current theory.

If condition (8), but not condition (7), is an equality then we have a corner solution where:

$$\lambda_i = \frac{\partial V}{\partial t_{N_i}} > \frac{\partial V}{\partial t_{C_i}}$$

The scientist would only spend time discovering a new theory.

What remains is to show that the model satisfies the two criteria for evaluating models of science. Recall that the first criterion was that the model be consistent with the rationality of science.

In the model presented here the progress of science has been defined as increasing values of a function (f) of the scope and the elegance of a theory. The actual act of acceptance of a new theory does not contribute directly to the advance or stagnation of science. Rather the act that influences progress is that of allocating time among scientific activities. Even though the acceptance of theories does not appear as an explicit factor in the progress of science, it is still possible to use acceptance as a measure of the rationality of science. The 'rationality' of science means that there must be objective criteria for judging theories. The substantive requirement for such criteria is that they be consistent with the functions of science. To fulfill the functions of science with ever-increasing effectiveness, it is necessary both that science itself continue to advance, and that what is best in current science be effectively promulgated. When a scientist 'accepts' a theory he is affirming that the theory should be promulgated because it is currently the most effective at fulfilling the functions of science. Thus the rationality of science requires the acceptance of the theory with the highest value of the function f which includes scope and elegance as independent variables. The acceptance of a theory does not mean that a scientist will spend all of his time trying to improve the theory he 'accepts'. Rather it means that for the function of effective

mental information processing and the function of promoting technology, he 'accepts' the theory as the best that science currently has to offer.

During periods in which a new theory is replacing an old one, scientists will differ in the speed of their recognition that the new theory has in fact become superior. The speed of acceptance will depend on how much information the scientist has on the scope and elegance of new theories. The extent of his information will, in turn, generally depend on the extent to which he himself has been productive in adding scope and elegance to new theories (i.e., the quantity of his N). Thus the speed of acceptance of a new theory is an increasing function of N . Formally:

$$(10) \quad A_i = \varphi \left(\sum_{j=0}^i (N_j) \right)$$

where A_i is the probability of accepting a new, eventually superior theory in period i . *Ceteris paribus*, any factor that either increases the time spent in discovery or increases the productivity of that time, will increase the speed of acceptance of a new theory.

Here I will only sketch the mathematical technique used to learn the effects of changes in the parameters on the amount of time spent in discovery. When the optimal values of the choice variables ($t_{N_i}^*$ and $t_{C_i}^*$) are obtained from first order conditions for maximizing the Lagrangean, these optimal values may be substituted back into the first order conditions. The identities that result may then be differentiated with respect to the parameters (γ , δ , β , ε and r). Using Cramer's method, the effect of a change in parameter value on the value of $t_{N_i}^*$ or $t_{C_i}^*$ may be obtained. For more mathematical details see the appendix or, more generally, Silberberg, 1978, pp. 164–167. The results for the effect of changes in the parameters on the optimal quantity of time spent on new theories are:

$$(11) \quad \frac{\partial t_N^*}{\partial \gamma} \geq 0$$

$$(12) \quad \frac{\partial t_N^*}{\partial \delta} \leq 0$$

$$(13) \quad \frac{\partial t_N^*}{\partial \beta} \leq 0$$

$$(14) \quad \frac{\partial t_N^*}{\partial \varepsilon} \leq 0$$

$$(15) \quad \frac{\partial t_N^*}{\partial r} \geq 0$$

These results are not surprising. Inequality (11) says that an increase in the productivity of time spent in discovery will result in more time spent in discovery. Inequalities (12), (13) and (14) say that an increase in the productivity of time spent on adding to the dominant theory's scope and elegance, will result in less time spent in discovery. Finally, inequality (15) says that an increase in the rate of depreciation of knowledge of the current theory will result in more time spent in discovery.

In my model, differential acceptance by age is possible even though relativism avoiding rationality is built into the utility function. Thus, unlike Kuhn, I can account for the stylized fact without conceding the rationality of science. The possibility in my model of differential acceptance by age is derived as follows. Equation (10) specifies that the probability of acceptance is a function of the fruits of 'discovery':

$$(10) \quad A_i = \varphi \left(\sum_{j=0}^i (N_j) \right)$$

By equation (3) we know that the fruits of 'discovery' are a function of the time spent in discovery. The relevant first-order condition for a maximum in the case of an interior solution is:

$$(16) \quad \frac{\partial L}{\partial t_{C_i}} = \frac{\partial V}{\partial f} \frac{\partial f}{\partial t_{C_i}} - \lambda_i = 0$$

By adding λ_i to both sides and taking the derivative of f with respect to t_{C_i} , we obtain:

$$(17) \quad \lambda_i = \frac{\partial V}{\partial f} \left[\delta t_{C_i}^{\varepsilon-1} \left(\beta + \varepsilon \sum_{j=0}^i \left(\frac{t_{C_j}}{(1+r)^j} \right) \right) + \left(\sum_{k=i+1}^{\infty} t_{C_k}^{\varepsilon} \varepsilon \left(\frac{1}{(1+r)^{k-i}} \right) \right) \right]$$

Equation (17) implies two separate effects of a scientist's age on the quantity of time he spends on the current theory. As the scientist ages, the left-hand term in brackets will become larger while the right-hand term in brackets will become smaller. The left-hand term becomes larger

with age because the scientist has accumulated human capital in the current theory and hence is more productive at extending it. The right-hand term becomes smaller with age because the beneficial future effects of time spent now in extending the current theory, diminish as the number of remaining periods declines. If the left-hand term dominates, then the marginal utility of time spent on the current theory will increase with age, resulting in older scientists spending more time on the current theory and less time on discovering a new theory. If the right-hand term dominates, then the opposite will occur. The model thus illustrates how Planck's principle could be true, even though scientists were rational in the sense of valuing the scope and elegance of theories.

REVIEW OF THE EVIDENCE

Work in the Merton tradition of sociology of science has in the last twenty years vastly increased our systematic knowledge of the workings of scientific institutions (Cole and Cole, 1973; Zuckerman, 1977; Stigler and Friedland, 1975, 1979). But although much has been learned about the determinants and correlates of success in paper-writing, citations, and professional advancement, little has yet been done to systematically increase our knowledge of the determinants and correlates of theory acceptance. Instead, discussions of theory acceptance usually have taken the form either of abstract generalizations or else of detailed narrative case studies. Zuckerman's 1979 article provides a good survey of much of this work.

A more rigorous and systematic approach to understanding theory acceptance can be found in some recent work that tested the importance of a scientist's age (Hull *et al.*, 1978; Diamond, 1980; Gieryn and Hirsh, 1983). In the paper by Hull and his collaborators, two sorts of tests were performed to see whether age influenced the acceptance by British scientists of evolution in the ten years following the publication of *The Origin of the Species*. In the first test the authors found that for the fifty scientists who accepted evolution within ten years, the speed of acceptance was not related to age. The second test was performed with the whole sample of 67, including those who still had not accepted evolution as of 1869. For this sample the authors obtained maximum likelihood estimates of a logit to learn the effect of year of birth on the probability

of acceptance. The coefficient on year of birth was estimated to be 0.046 with a *t*-statistic of 2.159 and a coefficient of determination for the regression of 0.06. Thus, while age was statistically significant in the direction predicted by Planck, it explained less than ten percent of the variation in acceptance.

In another paper, I tested the importance of age in explaining the acceptance by economic historians of rigorous economic theory as applied to history (known as 'cliometrics'). Using a sample of the first 117 economic historians from the directory of the Economic History Association, I labelled all those as cliometricians who had been on a mailing list for a national cliometrics meeting. The intent of the organizer of the meeting had been to inform every academic whom he judged to have accepted cliometrics. As in the second evolution test, a logit regression was performed using acceptance as the dependent variable and year of birth as the independent variable. The coefficient on year of birth was estimated to be 0.024 with a *t*-statistic of 2.155 and a coefficient of determination for the regression of 0.07. The results were thus remarkably similar to the evolution test: the year of birth variable, although statistically significant in the direction predicted by Planck, explained less than ten percent of the variation in acceptance.

A study by Gieryn and Hirsh used a sample of 98 American scientists who authored or co-authored 7 or more articles in the field of X-ray astronomy over the period 1960–1975. "Innovations" were "defined as contributions that challenge dominant theoretical, empirical or technical assumptions, and which result in palpable changes in problems chosen for research, theories adopted and techniques exploited" (p. 93). On the basis of the definition, Hirsh classified papers as either innovative or non-innovative. A scientist was classified as innovative if his name appeared on one or more of the articles classified as innovative. Thirty-four of the 98 scientists were classified as innovative. The authors classified their sample by dichotomous career characteristics and applied chi-squared tests to test the significance of differences in percent of innovators for each characteristic. Of six such tests, only two were reported to be statistically significant. 52.4% of the 42 scientists who received their highest degree after 1965 were innovators while only 22.7% of the 44 scientists who received their highest degrees before 1965 were innovators. The second statistically significant finding was that 66.7% of

the 15 scientists who listed X-ray astronomy as one of their specialties were innovators while only 23.5% of the 17 scientists who did not report X-ray astronomy as one of their specialties were innovators.

Systematic analysis of data from a few episodes thus confirms the stylized fact that had been casually observed by many: older scientists accept new theories more slowly than do younger scientists.

CONCLUSION

In the past twenty years acute students of science have observed that scientists often disagree. From this they inferred that the values of science are changing and subjective. The inference is false. A maximization-under-constraints model of science shows that scientists may disagree about theories even though they agree on values. Such a model is able to account for science's rationality without denying science's history.

In the future the research begun here can be extended in several different directions. The present model could be reformulated with greater mathematical rigor using the analytical tools of calculus of variations or optimal control (e.g., Ben-Porath, 1967). Here I have basically modelled science as a consumption activity: scientists do science because they receive utility from advancing science. At least for comparison purposes it might be useful to develop a supply and demand model that had scientists maximizing income and other sectors of the economy demanding scientific knowledge. The important question of the optimal reward structure of science might be better addressed by such a model than by the model presented here.

As a preliminary to future modelling, time and effort should be allocated toward establishing a richer body of stylized facts in the tradition of Brahe, Burns and Mitchell (Koopmans, 1947). We do not know, for instance, whether younger scientists are quicker to accept new mistakes just as they are quicker to accept new advances. Research currently being conducted on the acceptance of polywater should provide the first systematic evidence on this issue (Diamond, 1986a, 1986b).

APPENDIX

In this appendix we derive in more mathematical detail the results (11)–(15) on pp. 157–158 that represented the effects of changes in the parameters on the optimal quantity of time spent on new theories. Our procedure here will be first to solve the problem for the case of an interior solution and then to modify that solution in the light of the Kuhn-Tucker conditions.

The Lagrangean method is first used to obtain the optimal values of t_{N_i} and t_{C_i} . The Lagrangean function is:

$$L = \sum_{i=0}^{\omega} [V_i + \lambda_i(T_i - t_{N_i} - t_{C_i})]$$

In what follows we suppress the period subscripts in order to avoid notational clutter. The first order conditions for a maximum are:

$$\frac{\partial L}{\partial t_N} = \frac{\partial V}{\partial t_N} - \lambda = 0$$

$$\frac{\partial L}{\partial t_C} = \frac{\partial V}{\partial t_C} - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = T - t_N - t_C = 0$$

If a unique solution exists, the three conditions can be solved for the three choice variables: t_N , t_C , and λ .

The optimal values t_N^* , t_C^* , and λ^* may then be substituted back into the three first order conditions in order to produce three identities. Define α as the vector of parameters (v , δ , β , ε , r). Then the three first order identities may be differentiated with respect to α and expressed in matrix form as:

$$\begin{pmatrix} L_{nn} & L_{nc} & -1 \\ L_{cn} & L_{cc} & -1 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial t_N^*}{\partial \alpha} \\ \frac{\partial t_C^*}{\partial \alpha} \\ \frac{\partial \lambda^*}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} -L_{n\alpha} \\ -L_{c\alpha} \\ -L_{\lambda\alpha} \end{pmatrix}$$

where

$$L_{nn} = \frac{\partial^2 L}{\partial t_N^{*2}}, L_{cn} = \frac{\partial^2 L}{\partial t_C^* \partial t_N^*}, \text{ etc.}$$

By the standard assumption of diminishing returns it follows that $L_{nn} < 0$ and $L_{cc} < 0$.

By the use of Cramer's rule we may then solve for $\partial t_N^*/\partial \gamma$, $\partial t_N^*/\partial \delta$, $\partial t_N^*/\partial \beta$, $\partial t_N^*/\partial \varepsilon$, and $\partial t_N^*/\partial r$. In what follows we make use of the fact that the bordered Hessian matrix, represented by H , is positive due to the assumption of diminishing returns on the utility function.

$$\frac{\partial t_N^*}{\partial \gamma} = \frac{\begin{vmatrix} -L_{n\gamma} & L_{nc} & -1 \\ -L_{c\gamma} & L_{cc} & -1 \\ -L_{\lambda\gamma} & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -L_{nn} & L_{nc} & -1 \\ L_{cn} & L_{cc} & -1 \\ -1 & -1 & 0 \end{vmatrix}} = \frac{\begin{vmatrix} -(+) & ? & -1 \\ 0 & - & -1 \\ 0 & -1 & 0 \end{vmatrix}}{H} > 0$$

$$\frac{\partial t_N^*}{\partial \delta} = \frac{\begin{vmatrix} -L_{n\delta} & L_{nc} & -1 \\ -L_{c\delta} & L_{cc} & -1 \\ -L_{\lambda\delta} & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -L_{nn} & L_{nc} & -1 \\ L_{cn} & L_{cc} & -1 \\ -1 & -1 & 0 \end{vmatrix}} = \frac{\begin{vmatrix} 0 & ? & -1 \\ -(+) & - & -1 \\ 0 & -1 & 0 \end{vmatrix}}{H} < 0$$

$$\begin{aligned}
\frac{\partial t_N^*}{\partial \beta} &= \underbrace{\begin{vmatrix} -L_{n\beta} & L_{nc} & -1 \\ -L_{c\beta} & L_{cc} & -1 \\ -L_{\lambda\beta} & -1 & 0 \end{vmatrix}}_H = \underbrace{\begin{vmatrix} 0 & ? & -1 \\ -(+) & - & -1 \\ 0 & -1 & 0 \end{vmatrix}}_+ < 0 \\
\frac{\partial t_N^*}{\partial \varepsilon} &= \underbrace{\begin{vmatrix} -L_{n\varepsilon} & L_{nc} & -1 \\ -L_{c\varepsilon} & L_{cc} & -1 \\ -L_{\lambda\varepsilon} & -1 & 0 \end{vmatrix}}_H = \underbrace{\begin{vmatrix} 0 & ? & -1 \\ -(+) & - & -1 \\ 0 & -1 & 0 \end{vmatrix}}_+ < 0 \\
\frac{\partial t_N^*}{\partial r} &= \underbrace{\begin{vmatrix} -L_{nr} & L_{nc} & -1 \\ -L_{cr} & L_{cc} & -1 \\ -L_{\lambda r} & -1 & 0 \end{vmatrix}}_H = \underbrace{\begin{vmatrix} 0 & ? & -1 \\ -(-) & - & -1 \\ 0 & -1 & 0 \end{vmatrix}}_+ > 0
\end{aligned}$$

The results expressed above need to be modified to take account of the Kuhn-Tucker conditions. In the case of $t_N^*=0$ the marginal value of time spent in discovery may remain below λ even when the parameters change. Hence each of the inequalities derived above may be equal to zero, yielding the form of the results expressed in (11)–(15) on pp. 157–158.

NOTES

* I am especially indebted for extensive comments from Gary Becker, Daniel Garber, Gary Kahn, Luis Locay, Donald McCloskey, George J. Stigler, Stephen Toulmin and William Wimsatt. The first version of the argument presented here was my Ph.D. dissertation. I received useful comments on an earlier version of this paper from Neil de Marchi, A. W. Coats, Mark Blaug and others at the 1984 History of Economic Society meetings.

¹ Max Planck, *Scientific Autobiography and Other Papers* (New York: Philosophical Library, 1949), pp. 33–34. In a less-cited passage, Planck gives a slightly different formulation to his principle: “An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: it rarely happens that *Sau* becomes *Paul*. What does happen is that its opponents die out and that the growing generation is familiarized with the idea from the beginning: another instance of the fact that the future lies with youth.” (Max Planck, *The Philosophy of Physics* [New York: W.W. Norton and Co., Inc., 1936], p. 97.)

² Cf. Kuhn: “... Aristotle’s subject was change-of-quality in general, including both the fall of the stone and the growth of a child to adulthood. In his physics, the subject that was to become mechanics was at best a still-not-quite-isolable special case.” (Kuhn, *The Essential Tension*, p. xi.) Cf. also Toulmin, *Human Understanding*, pp. 386–387.

³ Quine gives original credit for showing this to Sheffer. Peirce discovered the only other single connective that is by itself adequate. (Cf. Willard Van Ormen Quine, *Mathematical Logic* [New York: Harper and Row Publishers, 1962], pp. 45–49.)

⁴ Stephen Fretwell has used the budget-line/indifference-curve apparatus to argue that in ecology the behavior of scientists that maximizes prestige is *not* the behavior that maximizes the advance of science. (Stephen D. Fretwell, *Populations in a Seasonal Environment* [Princeton: Princeton University Press, 1972], pp. x–xvii).

⁵ Merton has argued that instead of property in ideas, scientists have property in priority of discovery of ideas. See: Robert K. Merton, ‘Priorities in Scientific Discovery’, in *The Sociology of Science* (Chicago: The University of Chicago Press, 1973), pp. 286–324.

⁶ Some evidence suggests that as a scientist ages, time spent in research declines and time spent in administration and gate-keeping activities increases (Diamond, 1986c). So in future work a more complex model might have T_i decline with age or, better yet, be determined endogenously.

REFERENCES

- Becker, Gary S.: 1976, ‘Altruism, Egoism and Genetic Fitness: Economics and Sociobiology’, in *The Economic Approach to Human Behavior*, Chicago: The University of Chicago Press.
- Becker, Gary S.: 1976, *The Economic Approach to Human Behavior*, Chicago: The University of Chicago Press.
- Ben-Porath, Yoram: 1967, ‘The Production of Human Capital and the Life Cycle of Earnings’, *Journal of Political Economy* **75**, 352–365.
- Cole, Jonathan R. and Stephen Cole: 1973, *Social Stratification in Science*, Chicago: The University of Chicago Press.
- Diamond, Arthur M., Jr.: 1978, ‘Science as a Rational Enterprise’, Ph.D. dissertation, Department of Philosophy, The University of Chicago.
- Diamond, Arthur M., Jr.: 1980, ‘Age and the Acceptance of Cliometrics’, *Journal of Economic History* **40**, 838–941.
- Diamond, Arthur M., Jr.: 1982, ‘Stable Values and Variable Constraints; the Sources of Behavioral and Cultural Differences’, *Journal of Business Ethics* **1**, 49–58.
- Diamond, Arthur M., Jr.: 1984, ‘An Economic Model of the Life-Cycle Research Productivity of Scientists’, *Scientometrics* **6**, 189–196.

- Diamond, Arthur M., Jr.: 1986a, 'The Career Consequences for a Scientist of a Mistaken Research Project', Xerox Draft, University of Nebraska at Omaha.
- Diamond, Arthur M., Jr.: 1986b, 'The Determinants of a Scientist's Choice of Research Projects', Xerox Draft, University of Nebraska at Omaha.
- Diamond, Arthur M., Jr.: 1986c, 'The Life-Cycle Research Productivity of Mathematicians and Scientists', *The Journal of Gerontology* **41**, 520-525.
- Fretwell, Stephen D.: 1972, *Populations in a Seasonal Environment*, Princeton: Princeton University Press.
- Gieryn, Thomas F. and Richard F. Hirsh: 1983, 'Marginality and Innovation in Science', *Social Studies of Science* **13**, 87-106.
- Grubel, Herbert G. and Lawrence A. Boland: n.d., 'On the Efficient Use of Mathematics in Economics: Some Theory, Facts and Results of an Opinion Survey'.
- Hall, A. Rupert: 1980, *Philosophers at War: The Quarrel Between Newton and Leibniz*, Cambridge: Cambridge University Press.
- Hull, David L.: 1978, 'Altruism in Science: A Sociobiological Model of Cooperative Behavior among Scientists', *Animal Behavior* **26**, 685-697.
- Hull, David L., Peter D. Tesser, and Arthur M. Diamond: 1978, 'Planck's Principle', *Science* **202**, 717-723.
- Klamer, Arjo: 1983, *Conversation with Economists*, Totowa, N. J.: Rowman & Allanheld.
- Koopmans, Tjalling C.: 1947, 'Measurement without Theory', *Review of Economics and Statistics* **29**, 161-172.
- Kuhn, Thomas: 1962, *The Structure of Scientific Revolutions*, Chicago: The University of Chicago Press.
- Kuhn, Thomas: 1977, *The Essential Tension*, Chicago: University of Chicago Press.
- Laudan, Larry: 1977, *Progress and Its Problems*, Berkeley: University of California Press.
- McCloskey, Donald N.: 1986a, *The Rhetoric of Economics* (Vol. 1 in the Series on the Rhetoric of the Human Sciences), Madison: University of Wisconsin Press.
- McCloskey, Donald N.: 1986b, 'Thick and Thin Methodologies in the History of Economic Thought' (April 1986b draft of paper to appear in Neil de Marchi and Wim Driehuis, eds., *The Popperian Legacy in Economics, and Beyond. A Symposium in Honor of J. J. Klant*, Cambridge University Press, forthcoming).
- Merton, Robert K.: 1973, 'Priorities in Scientific Discovery', in *The Sociology of Science in Europe*, Carbondale: Southern Illinois University Press.
- Planck, Max: 1936, *The Philosophy of Physics*, New York: W. W. Norton and Co., Inc.
- Planck, Max: 1949, *Scientific Autobiography and Other Papers*, New York: Philosophical Library.
- Quine, Willard Van Ormen: 1962, *Mathematical Logic*, New York: Harper and Row Publishers.
- Sarkar, Husian: 1983, *A Theory of Method*, Berkeley: University of California Press.
- Silberberg, Eugene: 1978, *The Structure of Economics*, New York: McGraw-Hill Book Co.
- Smith, Adam: 1979, *The Theory of Moral Sentiments*, Oxford: Clarendon Press (first edition was published in 1759).
- Stigler, George J. and Claire Friedland: 1975, 'The Citation Practices of Doctorates in Economics', *Journal of Political Economy* **83**, 477-507.
- Stigler, George J. and Claire Friedland: 1979, 'The Pattern of Citation Practices in Economics', *History of Political Economy* **11**, 1-20.
- Stove, David: 1982, *Popper and Beyond*, Oxford: Pergamon Press.

- Toulmin, Stephen: 1972, *Human Understanding*, Princeton, N.J.: Princeton University Press.
- Westfall, Richard S.: 1980, *Never at Rest: A Biography of Isaac Newton*, Cambridge: Cambridge University Press.
- Wilson, Edward O.: 1975, 'Group Selection and Altruism', in *Sociobiology*, Cambridge, Mass.: Harvard University Press.
- Zuckerman, Harriet: 1979, 'Theory Choice and Problem Choice in Science', Ch. 4 in J. Gaston (ed.), *Sociological Perspectives on Science*, San Francisco: Jossey-Bass.

Department of Economics,
University of Nebraska at Omaha,
Omaha, NE 68182-0048,
U.S.A.